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ON INCREMENTAL METHODS

FOR TRAFFIC ASSIGNMENT

by

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ABSTRACT

The purpose of this note is a critical appraisal of incremental methods for traffic assignment. The main point which we develop is that incremental methods do not solve the minimization problem associated with computing an assignment that satisfies Wardrop's first extremal principle. We illustrate our conclusion with a simple counter example.

Introduction

The purpose of this note is a critical appraisal of incremental methods for the traffic assignment. Martin and Manheim (1965) were the first to propose such an approach, in an attempt to improve on the traffic assignment techniques of the time. The techniques available then for restrained assignments were mostly based on heuristic variations on the "all or nothing" principle (such as Mosher, 1963) or the "diversion" techniques of Irwin and Von Cube (1962) and Overgaard (1967). The work of Martin and Manheim was an important step in the direction of computing an assignment that satisfies Wardrop's (1952) first extremal principle. The main point which we will present is that the incremental method proposed by Martin and Manheim and implemented in the computer package entitled ICES-DODOTRANS (see Manheim and Ruiter, 1970), does not solve the minimization problem associated with finding the equilibrium flows. The curious and interesting conclusion that we shall develop is that the technique may be modified to approximately solve single commodity minimum convex cost flow problems, and is similar to an algorithm of Busacker and Gowen (1961), however cannot be suitably modified for the multi-commodity problem.

It has come to our attention that the incremental technique is often used in practice and it may be important to give a rigorous evaluation of the technique. For ease of presentation, we shall describe the incremental assignment procedure for the fixed demand case only, although it has been developed for elastic travel demand as well.

Notation and Problem Formulation

We consider a transportation network of nodes N and oriented links A. A node represents either a zone (centroid) or an intersection of streets. A two way street is represented by two directed links of opposite orientation. The number of trips from origins p to destinations q are known and denoted g_{pq}. The link volume delay functions, $s_a(v_a)$, $a \epsilon A$, are convex increasing functions of the link flows v_a. On the network, g_{pq} subdivides into path flows $h_{k,pq}$. As usual we require that the flows be conserved

$$(1) \quad \sum_k h_{k,pq} = g_{pq}, \quad \text{all } (p,q)$$

and the link flows are related to path flows as

$$(2) \quad v_a = \sum_p \sum_q \sum_k \delta_{ak,pq} \cdot h_{k,pq}, \quad \text{all } a$$

where

$$\delta_{ak,pq} = \begin{cases} 1 & \text{if } a \text{ belongs to path } k \text{ for pair } (p,q) \\ 0 & \text{otherwise} \end{cases}$$

Also,

$$(3) \quad v_a \geq 0, \quad h_{k,pq} \geq 0.$$

Although the characterization of driver's behavior on a road network has not been definitively established, Wardrop's principles have become well accepted in the modeling of traffic flow. His first principle states that "The journey time on all routes actually used are equal, and less than those which would be experienced by a single

vehicle on any unused route". Intuitively this corresponds to the situation when no driver can reduce his journey time by deviating from his chosen route. Mathematically, this principle may be expressed as

$$(4) \quad \sum_a \delta_{ak,pq} \cdot s_a(v_a) = u_{pq} \quad \text{if} \quad h_k > 0$$

$$(5) \quad \sum_a \delta_{ak,pq} \cdot s_a(v_a) \geq u_{pq} \quad \text{if} \quad h_k = 0$$

The implicit assumption made is that the travel time on a path is the sum of the link times that make up the path. Now, if these equations are combined with the conservation of flow conditions (1) and the corresponding nonnegativity constraints (3) they constitute a quantitative statement of Wardrop's first principle. These conditions may be interpreted as the Kuhn Tucker conditions for an equivalent minimization problem which is

$$(6) \quad \text{Min} \sum_a \int_0^{v_a} s_a(x)\, dx$$

Subject to constraints (1), (3) as has been done in Beckmann *et al.* (1956).

The Incremental Algorithm and Some Deficiencies

Although several variations have been suggested on the incremental algorithm (see Van Vliet, 1973, for example), all such methods start with all arc flows being zero and gradually assign increments of the O-D interzonal trips on to the network until the flows are in a state of approximate

equilibrium. The detailed steps are as follows:

Step 1. Select an origin at random

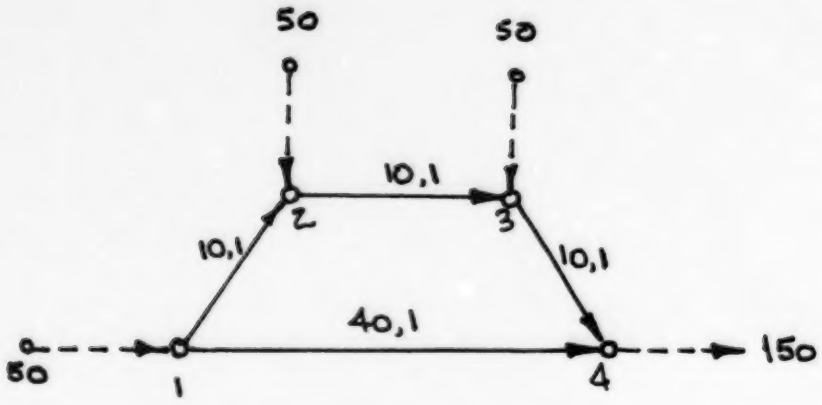
Step 2. Compute shortest paths to all destinations

Step 3. Assign to the routes found an increment of the difference between the total O-D trip demand and the trips already assigned

Step 4. If all the demand for all origins has been assigned, terminate, otherwise continue to

Step 5. Revise the path flows and link times - return to Step 1.

There are several inaccuracies that result from the above procedure. The size of the increment is an arbitrary choice, and intuitively it seems clear that the smaller the increment the closer to equilibrium will the resulting flow pattern be. Thus, there is an incentive to use as small an increment as possible, say 1 trip at a time. Clearly, such a small increment is prohibitive computationally when a large number of trips must be assigned. However, by far, the more serious deficiency of the method is that once assigned, an increment of flow cannot be reassigned to another path. The following example illustrates the discrepancies that may result due to this characteristic of the method. Consider the example of Figure 1



Link costs are $s_a(v_a) = c + d\,v_a$

(v_1, v_2) are indicated on the links

Figure 1

The equilibrium assignment is given in Figure 2

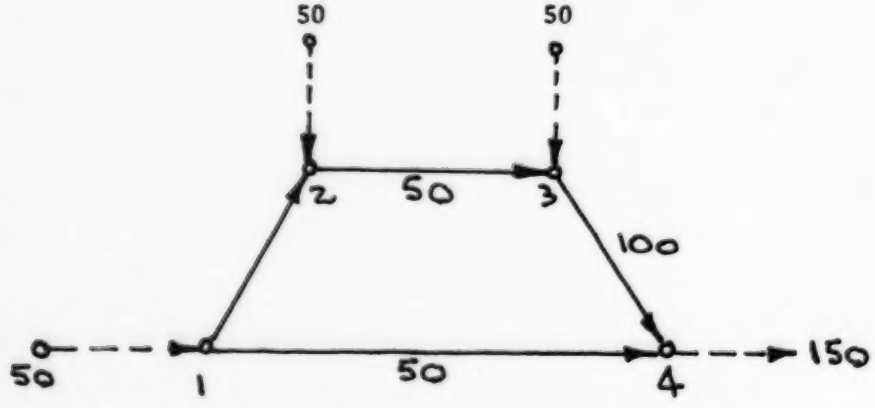


Figure 2

However, if one were to apply the incremental assignment method and node 1 were the first origin selected, then the first increment to be assigned would be a fraction of the 50 trips from 1 to 4 on the

path $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$. In any subsequent iterations this assignment would not be reassigned and the resulting flow pattern will be different than the equilibrium flow pattern.

In a recent unpublished M.Sc. thesis, Nestle (1974) carried out several experiments by the use of DODOTRANS I and found that the assignments obtained are quite sensitive to the order in which the O-D pairs are selected and to the increment used. We believe that the reason for his findings lies in the arguments presented above.

Discussion

We would like to further clarify the deficiencies of the incremental method for traffic assignment and show that it can be classified as a heuristic approach that unfortunately lacks the rigor of methods for traffic assignment suggested by Bruynoogue *et al.* (1968), Dafermos and Sparrow (1969), Dafermos (1971) and more recently by Nguyen (1974a, 1974b).

Consider a flow problem with convex costs such as (6), (1), (3) for a single origin-destination pair. This is then a single commodity flow problem which may be solved by the incremental (or dual) method of Busacker and Gowen. This method has the following steps:

Step 0. Start with all link flows equal to zero.

Step 1. Define new link costs as follows:

(7) $\bar{s}_a = s_a(v_a)$, for all links a if $v_a > 0$

Define a reverse link a', that is incident to the same nodes as a, but in opposite sense, with

(8) $\bar{s}_{a'} = -s_a(v_a-1)$, for all reverse links a'

Step 2. Compute a shortest path from the origin to the destination

Step 3. Revise the link flows as follows:

(9) $v_a = v_a+1$ for all links a on the shortest path

and $v_a = v_a-1$ for all links a such that the reverse link a' is on the shortest path.

Step 4. If all the demand has been assigned terminate. Otherwise return to Step 1.

The essential feature of this method, for our interest here, is that the addition of the reverse arcs with negative costs permits the reassignment of flow from one path to another, when the shortest path found in Step 1 employs some of these reverse arcs. The application of this algorithm results in a sequence of origin destination flows that are all minimal cost flows, until a flow having the desired value is reached.

Unfortunately the incremental method for the multi-commodity problem and therefore for traffic assignment cannot be based on an algorithm such as that outlined above. As the careful reader may convince himself by referring again to the counter example of Figures 1 and 2, it is not possible to incrementally load a multi-commodity flow on the network and maintain the equilibrium for all O-D pairs. Suppose that an increment of flow is added

for an O-D pair using the algorithm for a single commodity and then an increment for another O-D pair is added to the network by using the same algorithm. Even though the flow for the second O-D pair is added properly, the flow for the first O-D pair may no longer be a minimal cost flow or equivalently, is no longer an equilibrium flow. Thus, one cannot assign trips from various O-D pairs incrementally and guarantee that the equilibrium of a former O-D pair is not violated by successive O-D pairs.

Conclusion

We conclude that the incremental method for traffic assignment has undesirable properties and in general does not produce equilibrium flows. In our opinion a primal method such as given by Nguyen (1974a) is preferable and is not more costly or cumbersome to apply than an incremental method, even when the networks studied are very large.

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